AN INVENTORY MODEL FOR DETERIORATING ITEMS WITH VARIABLE TYPE DEMAND RATE AND DIFFERENT SELLING PRICES

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ABSTRACT

Many goods undergo decay or deterioration over time which suffer from depletion by direct spoilage while stored. So decay or deterioration of these goods in stock is a very realistic feature and it is necessary to use this factor in inventory models. In this paper we have developed an order level inventory model for constant rate of deterioration. We have also considered a variable type of demand which behaves differently in the given time horizon. The demand rate is constant for a certain fixed time and then the demand varies linearly with time. This paper also deals with different selling prices in two different time periods. The objective of the model is to find the optimal on-hand inventory by considering the profit function.

KEYWORDS: Deterministic Inventory Model, Deterioration, Profit functions, Two different selling prices, Variable demand rate

INTRODUCTION

One of the most developed fields of Operations Research is inventory modeling. Inventory has been defined as an idle resource that possess economic value by Monks(1987). Keeping an inventory for future sales or use is very common in business. Retail firms, wholesalers, manufacturing companies and even blood banks generally have a stock of goods on hand. Usually the demand rate is decided by the amount of the stock level. The motivational effect on the people may be caused by the presence of stock at times. Large quantities of goods displayed in markets according to seasons motivate the customers to buy more. If the stock is insufficient the customers may prefer some other brands, as shortages will fetch loss to the producers. On the other hand, deterioration is an important natural phenomenon and the consequent loss due to decay of items may be quite significant. Mainly when, physical goods are stocked for future use, in some items such as medicines, foodstuff, dairy items, volatile liquids, the process of deterioration is observed.
In the present paper demand rate is considered as constant to a fixed time and then it varies linearly with time. We have also considered different selling prices in two time periods since the demand decrease with time. It is assumed that lead time is zero and shortages are not allowed. The objective of the model is to find the on-hand inventory by maximizing the profit function.

**FUNDAMENTAL ASSUMPTIONS AND NOTATIONS**

Following assumptions are made for the proposed model:
- Demand rate is variable with respect to time.
- Single inventory will be used.
- Lead time is zero.
- Shortages are not allowed.
- Replenishment rate is infinite but size is finite
- Time horizon is finite.
- There is no repair of deteriorated items will occur during the cycle

Following notations are made for the given model:
- \( I(t) = \) On hand inventory at time \( t \)
- \( R(t) = \) Demand
- \( \theta = \) The constant deterioration rate where \( 0 \leq \theta \leq 1 \)
- \( I_0 = \) Inventory at time \( t=0 \)
- \( s_1 = \) Selling price per unit in \( (0,t_1) \)
- \( s_2 = \) Selling price per unit in \( (t_1,T) \)
- \( c = \) Unit cost of the item per unit time
- \( Q = \) On-hand inventory
- \( H = \) Holding cost per unit item per unit time
- \( r = \) Replenishment cost per replenishment which is a constant
- \( T = \) Duration of a cycle
- \( R_0 = \) Initial demand rate
- \( a = \) Rate of change of demand with respect to \( t \)

**DESCRIPTION OF THE MODEL**

In this model we consider the rate of demand \( R(t) \) to be a constant up to a certain time \( t=t_1 \) and after which it varies linearly with time. If \( I(t) \) be the on hand inventory at time \( t \geq 0 \), then at time \( t+\Delta t \), the

\[
I(t+\Delta t) = I(t) - R(t)\Delta t - \theta. I(t) \Delta t
\]

Dividing by \( \Delta t \) on both sides,

\[
\frac{I(t+\Delta t)}{\Delta t} - \frac{I(t)}{\Delta t} = -R(t) - \theta. I(t) \frac{\Delta t}{\Delta t}
\]

Taking limit on both sides as \( \Delta t \to 0 \),

\[
\lim_{\Delta t \to 0} \frac{I(t+\Delta t)}{\Delta t} = \lim_{\Delta t \to 0} \left( \frac{I(t)}{\Delta t} - \frac{R(t)\Delta t}{\Delta t} - \frac{\theta I(t)\Delta t}{\Delta t} \right)
\]

\[
= \lim_{\Delta t \to 0} (-R(t) - \theta. I(t))
\]

Define \( R(t) = R_0 + a(t-t_1). H(t-t_1) \) \( \quad \rightarrow (2) \)

where \( H(t-t_1) = 1 \) for \( t \geq t_1 \)
\( = 0 \) for \( t \leq t_1 \)

Now equation (1) becomes,

\[
\frac{di}{dt} = -R_0 - \theta. I(t), \text{if } 0 \leq t \leq t_1
\]

\[
= -R_0 - a(t-t_1) - \theta. I(t), \text{if } t_1 \leq t \leq T \quad \rightarrow (3)
\]

Take integral with respect to \( t \) in equation (3)

\[
\int \frac{di}{dt} dt = \int (R(t) + \theta. I(t)) dt
\]

\[
I(t) = \frac{(R(t) + \theta. I(t))}{\theta} + c
\]

\[
\rightarrow (4)
\]

When \( I = I_0 \) and \( t = 0 \),

\[
0 = \frac{-R_0}{\theta} - 0 + c
\]
\[
r = \frac{c}{R_0}
\]

Now (*) becomes

\[
I_0 = \left[ \frac{-R_0}{\theta} - I_0 \right] e^{-\theta t} + \frac{R_0}{\theta}
\]

\[
I = \frac{-R_0}{\theta} + e^{-\theta t} \left[ I_0 + \frac{R_0}{\theta} \right] \quad \rightarrow (5)
\]

Again using \( I = I_1 \) at \( t = t_1 \)

\[
I_1 = \frac{-R_0}{\theta} + e^{-\theta t} \left[ I_0 + \frac{R_0}{\theta} \right]
\]

\[
\theta = -R_0 + e^{-\theta t} \left[ Io \theta + R_0 \right]
\]

Taking log on both sides,

\[
\log I_1 \theta = \log(-R_0 + (-\theta t_1) + \log(Io \theta + R_0)
\]

\[
t_1 = -\frac{1}{\theta} \log \left( \frac{(I_0 \theta + R_0)}{(I_0 \theta + R_0)} \right) \quad \rightarrow (6)
\]

From equation (4),

\[
\frac{dt}{dt} = -R_0 - a(t-t_1) - \theta. I(t), \text{if } t_1 \leq t \leq T
\]

Taking integral with respect to \( t \) on both sides,

\[
I = \int R(t) dt - \int \left( a(t-t_1) + \theta. I(t) \right) dt
\]

\[
= -R_0 t - \int \left[ \frac{a(t-t_1) + \theta. I(t)}{\theta} \right] + c
\]
\[= - R_0 t \cdot \frac{a(t-t_1)}{a} \cdot I(t) + c \]

\[
\to (**) \]

To find \(c\) applying initial condition, \(I = I_0, t = T\)

Now \((**)*\) becomes,

\[I = \frac{a(T-t_1)}{a} \quad \text{for} \quad t_1 \leq t \leq T \]

when \(I = I_1\) at \(t = t_1\) we have

\[I_1 = \frac{a(T-t_1)}{a} \quad \text{since} \quad t_1 = -\frac{1}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \]

\[I_1 = \frac{aT}{a} + \frac{a}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \]

\[aT + \frac{a}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \]

\[\theta I_1 - \frac{a}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) = aT \]

Divide by \(a\) on both sides,

\[\frac{\theta I_1}{a} - \frac{a}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) = T \]

\[\theta I_1 + \frac{a}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) = T \]

Now average on hand inventory is given by,

\[Q = \int_0^T \frac{aT}{a} dt + \int_0^T I_1 \, dt \]

Now substituting the corresponding values of \(I\) we have,

\[Q = \frac{1}{a} \left[ I_0 + \frac{R_0}{\theta} \left( 1 - e^{-\theta t} \right) - \frac{R_0 t}{\theta} + \frac{a}{\theta} (T - t_1)^2 \right] \]

\[= \frac{1}{a} \left[ I_0 + \frac{R_0}{\theta} \left( 1 - \frac{10}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \right) \right. \]

\[\left. + \frac{a}{\theta} \left( \frac{\theta I_1}{a} - \frac{a}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \right) + \frac{1}{a} \left[ I_0 + \frac{R_0}{\theta} \right] \right] \]

\[= \frac{1}{a} \left[ I_0 + \frac{R_0}{\theta} \left( 1 - \frac{10}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) + \frac{R_0}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) + \frac{a}{20} \right] \]

\[= \frac{1}{a} \left[ I_0 + \frac{R_0}{\theta} \left( 1 - \frac{R_0}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) + \frac{a}{20} \right) \right. \]

\[= \frac{1}{R_0 + R_0 + \theta 0} \left( 1 - 11 \right) + \frac{R_0}{\theta^2} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) + \frac{1}{20} \]

\[\text{(c)} \]

Integrating the first part of \((c)\) we get

\[\int_0^{t_1} - \frac{R_0}{\theta} \left[ I_0 + \frac{R_0}{\theta} \right] dt \]

\[= - \frac{R_0}{\theta} \left[ I_0 + \frac{R_0}{\theta} \right] \left[ -\theta e^{-\theta t_1} \right] \]

\[= - \frac{R_0 t_1}{\theta} + \left[ I_0 + \frac{R_0}{\theta} \right] \left( - \theta e^{-\theta t_1} \right) \]

\[= \frac{R_0 t_1}{\theta} + \left[ I_0 + \frac{R_0}{\theta} \right] \left( 1 - e^{-\theta t_1} \right) \]

\[= \frac{1}{\theta} \left( 1 - e^{-\theta t_1} \right) \left[ I_0 + \frac{R_0}{\theta} \right] \]

Integrating the second part of the above equation \((c)\) we get

\[\int_{t_1}^T \frac{a}{\theta} (T - t)dt \]

\[= \frac{a}{\theta} \left[ T(T-t_1) - \frac{t_1^2}{2} \right] \]

\[= \frac{a}{\theta} \left[ T(T-t_1) - \frac{1}{2} \left( T^2 - t_1^2 \right) \right] \]

\[= \frac{a}{\theta} \left[ (T-t_1) \left( T - \frac{1}{2} T - t_1 \right) \right] \]

\[= \frac{a}{\theta} \left( T - t_1 \right)^2 \]

\[\cdot Q = \frac{1}{\theta} \left[ I_0 + \frac{R_0}{\theta} \right] \left[ 1 - e^{-\theta t_1} \right] \cdot \frac{R_0 t_1}{\theta} + \frac{a}{20} (T - t_1)^2 \]

\[= \frac{1}{\theta} \left[ I_0 + \frac{R_0}{\theta} \right] \left[ \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \right] \]

\[\text{The profit function, } \phi(I_0) \]

\[\phi(I_0) = \frac{1}{T} \left( s_1 + s_2 - c - r \right) I_0 - \frac{h}{T} \left( I_0 - \frac{R_0}{\theta} \right) + \frac{\theta T}{\theta} \]

\[= \frac{1}{T} \left( s_1 + s_2 - r \right) I_0 + \frac{\theta T}{\theta} + \frac{h R_0}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \]

\[= \frac{1}{T} \left( s_1 + s_2 - r \right) I_0 + \frac{h R_0}{\theta} \log \left( \frac{I_1 + R_0}{I_0 + R_0} \right) \]

\[\text{Differentiate with respect to } I_0 \text{ on both sides,} \]

\[\frac{d}{dI_0} (\phi(I_0)) = \frac{s_1 + s_2 - r}{T} - \frac{h}{T} \left( \frac{I_0}{\theta} + \frac{R_0}{\theta} \right) \]

\[= \frac{s_1 + s_2 - r}{T} - \frac{h}{T} \left( \frac{I_0}{\theta} + \frac{R_0}{\theta} \right) \]

\[= \frac{1}{T} \left( s_1 + s_2 - r \right) - \frac{h}{T} \left( \frac{I_0}{\theta} + \frac{R_0}{\theta} \right) \]

\[\text{The necessary condition for } (I_0) \text{ to attain maximum is} \]

\[\frac{d}{dI_0} (\phi(I_0)) = 0 \text{ which gives,} \]

\[I_0 = \frac{R_0 (s_1 + s_2 - r)}{h} \]

\[\text{Again differentiating with respect to } I_0 \text{ both sides} \]

\[\frac{d^2}{dI_0^2} (\phi(I_0)) = \frac{-h R_0}{I_0 + R_0} \left( \frac{1}{\theta^2} \right) \]

\[= \frac{-h R_0}{I_0 + R_0} \left( \frac{1}{\theta^2} \right) \]

\[\text{It will give a global maximum for profit function } (\phi(I_0) \]
REFERENCES


